How well do we know the electromagnetic form factors of the proton?

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Abstract. Recent measurements of recoil polarization in elastic scattering have been used to extract the ratio of the electric to the magnetic proton form factors. These results disagree with Rosenbluth extractions from cross-section measurements, indicating either an inconsistency between the two techniques, or a problem with either the polarization transfer or cross-section measurements. To obtain precise knowledge of the proton form factors, we must first understand the source of this discrepancy.

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1 Introduction

Elastic form factors measurements probe the charge and magnetization distributions of the nucleon, and provide strong constraints on models of nucleon structure. Prior to the year 2000, all of the high-Q^2 proton form factor data came from cross-section measurements, utilizing the Rosenbluth technique to separate the electric and magnetic form factors, G_E and G_M. A global analysis of the large body of data on elastic electron-proton scattering indicated that G_M follows the dipole form, $G_M = 1/(Q^2 + 0.71)^2$, with $\sim 5\%$ deviations (fig. 1). While the measurements of G_E at high Q^2 are significantly less precise, the extracted ratio of G_E to G_M is roughly constant.

More recent G_E/G_M results, from measurements of the polarization of the recoil protons, show that G_E falls more rapidly with Q^2 [1] . There are significant deviations from the global Rosenbluth analysis above $Q^2 = 1$ GeV^2, as shown in fig. 1. This discrepancy indicates that something is wrong with one of these two techniques, or one or more of the experiments. I will briefly review the two techniques, focusing on potential sources of systematic uncertainties. Next, I will give an overview of a recently completed JLab experiment designed to test the compatibility of the two techniques. Finally, I will present an analysis of the previous data designed to look for possible sources of the discrepancy between the two techniques. The goal is to determine what kind of errors would have to exist in the cross-section measurements to explain the discrepancies in the form factors ($\gtrsim 100\%$ error in G_E, few % in G_M) and
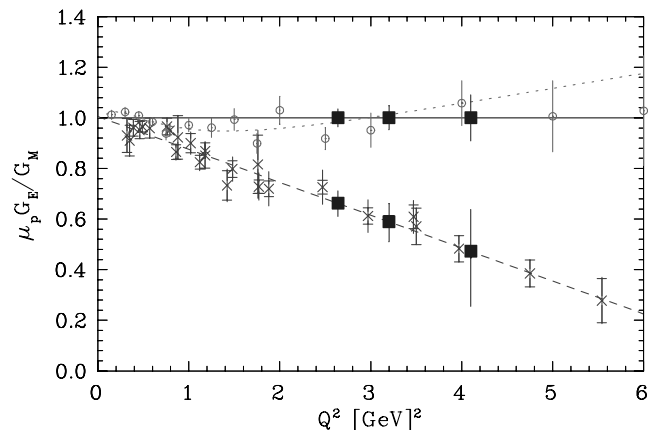
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Fig. 1. Ratio of $\mu_p G_E/G_M$ from a global analysis of cross-section data [2] (circles), and from recent polarization transfer measurements [1] (crosses). The curves are the fits [3,1] to the cross-section and polarization data. The squares are the projected uncertainties for E01-001 (sect. 3).

what impact these errors might have on our knowledge of the proton form factors, as well as other measurements.

2 Extractions of the elastic form factors

Rosenbluth extractions (L/T separations) of the form factors are performed by measuring elastic electron-proton scattering at fixed virtual-photon energy and momentum $(\nu, \mathbf{q})$, while varying the electron energy and scattering angle to vary the virtual-photon polarization, ϵ. The reduced cross-section, σ_R, can then be expressed in terms of

the form factors, which depend only on Q^2 ($= q^2 - \nu^2$):

$$\sigma_R \equiv \frac{d\sigma}{d\Omega} \frac{\epsilon(1+\tau)}{\sigma_{\text{Mott}}} = \tau G_M^2(Q^2) + \epsilon G_E^2(Q^2), \qquad (1)$$

where $\tau = Q^2/(4M_p^2)$. G_M is then extracted from the reduced cross-section at $\epsilon = 0$, while G_E is extracted from the ϵ-dependence. Due to the ϵ/τ weighting of the electric term relative to the magnetic term, the contribution from G_E decreases as $1/Q^2$ for a fixed ratio of G_E/G_M, and isolating the contribution of G_E becomes increasingly difficult as Q^2 increases. Because of this, it is important, and increasingly difficult as Q^2 increases, to ensure that the ϵ-dependent systematics do not overwhelm the uncertainties in the extraction. Because ϵ is correlated with beam energy, scattering angle, and scattered electron energy for a fixed value of Q^2, and because the Mott cross-section varies rapidly with angle at fixed Q^2, there are several potential sources of ϵ-dependent errors which might affect the extracted form factors.

An alternative technique involves using polarized electrons and measuring the polarization of the recoiling proton. The ratio of the transverse to longitudinal components of this polarization is directly related to G_E/G_M. Measuring a ratio of two polarization components means that uncertainties in the cross-section, beam polarization, and detector analyzing power all cancel out, significantly reducing the dominant sources of systematic uncertainty. While this method is clearly superior at large Q^2 values, the discrepancy between the Rosenbluth and recoil polarization measurements occur at Q^2 values as low as $\sim 1 \text{ GeV}^2$, where both techniques give precise measurements.

3 Jefferson Lab experiment E01-001

Jefferson Lab experiment E01-001 was designed to test the consistency of two techniques. In the global analysis of the cross-section measurements, a major concern is the relative normalization of the difference experiments. While a normalization factor for each experiment is determined from the best fit to the entire data sets, there is still room for the normalizations to vary, which could lead to a change in the extracted form factors. Single-experiment extractions eliminate the effect of normalization uncertainties, which can dominate the uncertainty in a global analysis. For the existing single-experiment L/T extractions, the dominant sources of uncertainty come from possible errors that could be correlated with ϵ. The largest such uncertainties come from rate-dependent corrections, as the Mott cross-section varies rapidly with scattering angle for fixed Q^2, and kinematic-dependent corrections, which may be especially important for extremely large or small values of the scattered electron momentum, where effects such as multiple scattering or non-linearities in magnetic spectrometers may become important. The goal of E01-001 was to make an extremely precise Rosenbluth extraction of G_E/G_M in a single measurement, with careful checks on systematic uncertainties. Data was taken at three Q^2 values from 2.6 to 4.1 GeV^2 to see if the two

techniques give consistent results for the ratio of G_E/G_M in the region where the previous L/T separations disagree with the new polarization transfer measurements.

E01-001 differs from previous experiments in two main respects: first, we measured the elastic cross-section by detection of the struck proton rather than the scattered electron, and second, we made simultaneous measurements at high and low Q^2 values for each beam energy. At fixed Q^2, the proton momentum stays fixed, and so there are no momentum-dependent corrections for the protons. In addition, the rate-dependence is dramatically reduced when detecting protons. For the kinematics of our experiment, the proton cross-section variation is $< 50\%$ over the full ϵ-range at each Q^2 value. If the electrons were detected at the corresponding kinematics, the cross-section would vary by 1-2 orders of magnitude between high and low values of ϵ. Finally, the cross-section is typically a factor of 2-4 less sensitive to uncertainties in beam energy and scattering angles, making the measurement less sensitive to small uncertainties in the scattering kinematics.

The main measurement is compared to a normalization point at low Q^2 (0.5 GeV^2), where the ratio of G_E/G_M is well known and, more importantly, where the ϵ-range of the measurement is very small ($\Delta\epsilon \approx 0.05$). Because ϵ is nearly constant for the low-Q^2 measurement, the reduced cross-section has a very small, and well known, ϵ-dependence, and we can use this data as a luminosity monitor to correct for the beam current, target thickness, and target density fluctuations.

The experiment was run in May 2002, and data was taken at $Q^2 = 2.64$, 3.2, and 4.1 GeV^2. The two lower Q^2 points will give the most precise results, and each should provide a better than 7σ differentiation between form factor scaling, $\mu_p G_E/G_M = 1$, and the decrease in G_E/G_M measured by the polarization measurements. The $Q^2 = 4.1$ point has larger uncertainties, due to both reduced statistics and increased background, but should still give a 4σ separation between the two results. The existing data are shown in fig. 1 along with projected results for E01-001 under two different assumptions for the extracted ratio. Because the rate-dependence and kinematic sensitivities are greatly reduced compared to previous measurements, the errors are dominated by uncertainties which are uncorrelated at the different Q^2 values, making the three measurements largely independent. In a typical L/T separation measurement, any rate-dependent or momentum-dependent errors would be likely to give similar effects for the extractions at all Q^2 values, and might change the overall trend of the data for all Q^2 points.

4 Global reanalysis of cross-section data

Because of the difficulties in performing L/T separations at high Q^2, where G_E contributes only a few percent to the cross-section, the recoil polarization technique is more reliable at large Q^2. However, the disagreement between the two techniques extends to $Q^2 \approx 1 \text{ GeV}^2$. In this range, the electric form factor contributes 20–30% to the cross-section at $\epsilon = 1$, and Rosenbluth separations give a precise
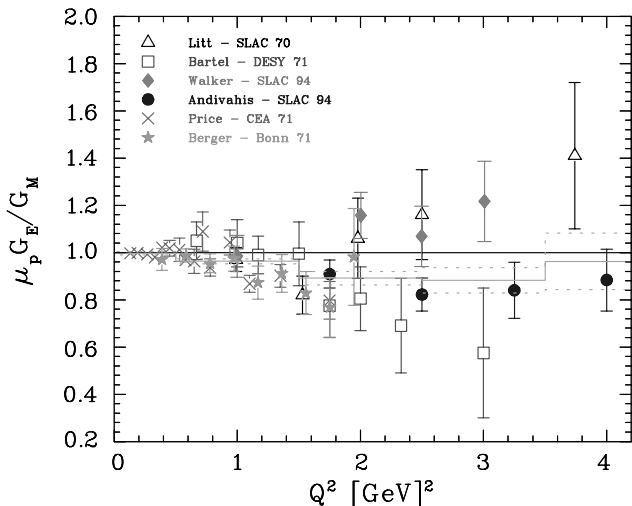
Fig. 2. $\mu_p G_E/G_M$ from individual Rosenbluth extractions.

measurement of G_E/G_M. As mentioned above, the experimental normalization factors and ϵ-dependent systematics are very important for these extractions, and it is possible that the disagreement is due to problems with some subset of the cross-section measurements in the global analysis. In fact, it has been noted that the results from different experiments that have extracted G_E are inconsistent at large Q^2 values. We will first present the individual Rosenbluth separations, and show that the inconsistencies between different data sets appear to be a combination of the assumptions in the analyses along with an error in one of the data sets. We then will try to determine if the disagreement between the two techniques can be explained by a simple problem with one or more data sets in the analysis, or any problems in the analysis itself.

4.1 Analysis of individual Rosenbluth measurements

Figure 2 shows the measurements of $\mu_p G_E/G_M$ for several different Rosenbluth separation measurements [2,4]. The data has been binned into five Q^2 bins, and the solid and dotted lines show the weighted average for each bin along with the 1σ uncertainties. While the combined data set shows approximate form factor scaling, with a decrease of $\sim 10\%$ at moderate Q^2 values, the individual measurements have significant scatter about this average. Comparing each data point to the average value for its Q^2 bin, we get $\chi_\nu^2 = 1.26$ for 40 degrees of freedom (*i.e.* a confidence level (CL) of 13%). The disagreement is more obvious if we focus on the high-Q^2 data: $\chi_\nu^2 = 1.63$ for 17 degrees of freedom for data above $Q^2 = 1.5$ GeV^2 (5% CL). The scatter of these results has been used to argue that the experiments are inconsistent, and that these results should be discarded.

Before concluding that the Rosenbluth extractions are not reliable, we should examine these data more carefully. There are two problems in this comparison of "single-experiment" extractions. First, the Walker data has a correction at small angles that was discovered by a later

SLAC experiment, but was not taken into account in the original analysis. Second, the other extractions shown in fig. 2 are not really single-experiment measurements. In three cases (Litt, Price, and Berger), the values of G_E and G_M come from combining a new set of cross-section measurements with older data at different ϵ values. Various procedures have been used to determine a relative normalization between the two experiments, but the uncertainties from this determination are either ignored altogether, or applied without taking into account the fact that adjusting the normalization of one data set leads to uncertainties that are highly correlated between the different Q^2 values. For the Bartel and Andivahis data, the form factors are extracted using only the new data, but that data comes from multiple data sets (taken using different spectrometers, or detecting protons rather than electrons). For these experiments, a direct measurement of the relative normalizations at identical kinematics was possible, and so the normalization factors should be better determined than in the previous cases, where the normalization factors had to be determined from data sets that did not have any kinematical overlap. However, while the normalization factors should be better determined, the correlated nature of the uncertainties from this determination was not taken into account. Thus, the extracted form factor ratios shown in fig. 2 are not a proper basis for determining the consistency of the different cross-section measurements.

While the ratios shown in fig. 2 are correct for the normalizations used in these analyses, the ratio will increase or decrease for all Q^2 values if these normalization factors are varied. Because the ratio is sensitive to the ϵ-dependence, and because this dependence decreases with Q^2, a shift in the normalization factors would have the largest effect at higher Q^2, and so each of these data sets could be caused to rise or fall with Q^2 by varying the normalization factors. While it may require a large change in the normalization correction, these analyses can all be adjusted to reproduce the falloff seen in the polarization transfer measurement (*e.g.*, for the most precise measurement, the Andivahis data, a 4% change in the measured correction for the low-ϵ data brings the ratio into agreement with the Hall A data, but at the cost of introducing a 6σ disagreement between the two spectrometers for measurements at identical kinematics). In order to study the consistency of the data, we must avoid the large uncertainties related to the normalization factors. To do this, we must either fully take into account the correlated uncertainties arising from the choice of normalization procedure, or else extract the form factors from single experiments, where these normalization issues do not arise. Alternatively, in a global analysis the normalization factors can be better determined, as the data sets will have significantly more overlap than in the case where two data sets, one at high ϵ and the other at low ϵ, are combined. This should lead to a more precise determination of the normalizations. We can then see if adjustments of the normalization factors within these uncertainties can significantly change the results. In the following sections, we will both analyze the single-experiment Rosenbluth extractions and
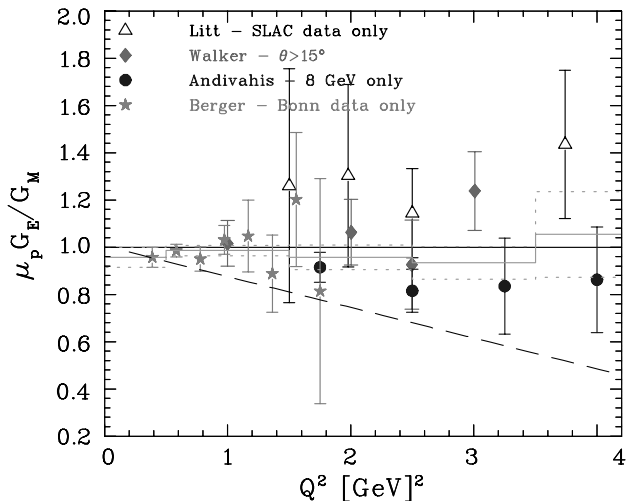
Fig. 3. $\mu_p G_E/G_M$ from individual Rosenbluth extractions using only data from single experiments. The dashed line is the fit to the Hall A recoil polarization measurements.

Fig. 4. $\mu_p G_E/G_M$ from various global fits: the dotted line is the fit to the Hall A data, the dashed line is the Bosted fit to the previous global analysis, the uppermost solid line is the new global fit to the cross-section data, the middle solid line is the fit with three data sets removed (see text), and the bottom solid line is the combined fit to the cross-section and recoil polarization data.

perform a global analysis to examine the discrepancy with the polarization measurements.

We begin by repeating the extraction of G_E/G_M for only those experiments where the ϵ-range was adequate to perform an L/T separation using the data from a single detector. For the Walker data, cross-sections taken below 15° were excluded to avoid the error from the missing correction. For the Andivahis data, we use only the data from the 8 GeV spectrometer, and exclude the 1.6 GeV spectrometer data (which always provided a single low-ϵ point). Of the other experiments, only the Litt and Berger data had enough ϵ-range to perform a stand-alone Rosenbluth extraction. The extractions from this limited data set are shown in fig. 3. The average again yields a ratio that is close to unity, but the data sets are now in better agreement: $\chi_\nu^2 = 1.18$ for 10 degrees of freedom (30% CL), for data above $Q^2 = 1.5$ GeV^2. The average is clearly well above the polarization data; in fact, all 20 data points lie above the Hall A fit.

4.2 Global fit to cross-section data

While these few stand-alone extractions are self-consistent, we would like to examine the full body of data to determine if the disagreement between the global analysis and the new polarization results can be at least partially explained by some problem in the data or analysis. The fit may be affected by inclusion of bad data points or data sets. It may even be that while the *best fit* yields a roughly constant ratio of G_E to G_M, adjustment of the normalization factors for the experiments may bring down this ratio at high Q^2 without significantly decreasing the overall quality of the fit. To test such explanations of the discrepancy between the two techniques, a new global analysis of the cross-section measurements is presented which can be used to test the above possibilities.
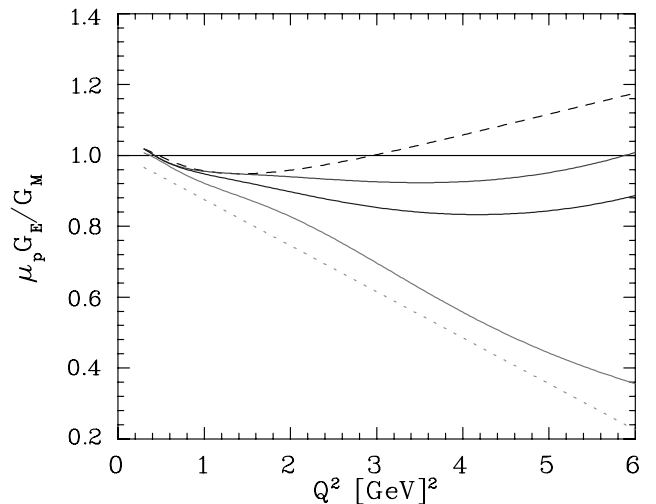
The global analysis is largely a repeat of the analysis performed in ref. [2]. We use the same data sets, and perform a combined fit to the electric and magnetic form factors (with $1/G_E$ and $1/G_M$ parameterized with 6th-order polynomials in $q = \sqrt{Q^2}$) along with a normalization factor for each of the data sets. However, there are some differences in the data sets and fitting procedures. For the Walker experiment, we exclude the data below 15 degrees, as discussed in the previous section. For the Andivahis measurement, we use the final published cross-sections, which were not available at the time of the previous global analysis. For the Andivahis and Berger experiments, we break up the data into subsets, one for each detector configuration. Thus, data taken from 13 experiments is broken up into 16 subsets, each with its own normalization constant. Finally, we exclude data below $Q^2 = 0.3$ GeV^2 and above $Q^2 = 10$ GeV^2, as we are mainly interested in the Rosenbluth results in the region where we have polarization transfer measurements: $0.5 < Q^2 < 6$ GeV^2. This initial fit gives similar results to the Bosted parameterization [3] of the Walker global analysis. The top two curves in fig. 4 show the Bosted fit (dashed line) and the result of the new fit (uppermost solid line).

Removal of the low-angle Walker data combined with breaking up the Andivahis and Bartel data sets slightly reduced the extracted ratio, but it is still just 5–10% below scaling, well above the parameterization of the Hall A data (dotted line). In examining the individual experiments, there are no data sets that have unacceptably large χ^2 values compared to the global fit, and no data point that lie beyond 3σ from the fit. These simple tests do not indicate any clear problems with the data sets. However, if a data set has an ϵ-*dependent* systematic uncertainty that is not too large compared to the uncorrelated

uncertainties, then this data set may bias the extracted ratio over a range in Q^2 without having an unusually large χ^2. In addition, the total χ^2 for the fit is quite low: $\chi^2 = 220.4$ for 274 degrees of freedom, indicating that some of the experiments have overestimated the uncertainties. Thus, the simple statistical test above may not be sufficient, and we need additional checks for possible bad data sets that could have a large impact on the overall result.

The first test involved repeating the fit 16 times, with a different data set removed each time. These fits had only very small changes in the extracted ratio. The fit was repeated, this time with the removal of the three data sets whose exclusion caused the largest reduction in the extracted ratio. Even this "worst-case" removal of three data sets leads to a small reduction ($\lesssim 10\%$) in the ratio (middle curve in fig. 4). Thus, if the discrepancy is caused by errors in the cross-section measurements, it must be a systematic, ϵ-dependent error that impacts several data sets rather than just a problem with one or two experiments.

Finally, we wish to see if small modifications to the normalization factors can remove the inconsistency between the two techniques without making the overall fit significantly worse. First, we perform a constrained fit to the data. We use the same 16 data sets as in our original fit, allowing G_M and the normalization factors for each experiment to vary, but requiring G_E to match the polarization data by constraining the ratio $\mu_p G_E/G_M = 1 - 0.13(Q^2 - 0.04)$. In this way, the normalization factors will be optimized to reproduce this ratio, as well as maximizing the consistency between the different data sets. If only small adjustments to the normalization factors are required, then the χ^2 for this fit should be only slightly worse than the unconstrained fit. When the ratio is constrained, the total χ^2 of the fit increases by 60.5 while the number of degrees of freedom increases by 6 (from 274 to 280). While the total χ^2 is still close to one, due to the overly conservative error estimates in some of the data sets, the increase in χ^2_ν is 0.20, which is extremely large for a fit to more than 300 data points. Constraining G_E/G_M to match the Hall A fit clearly gives too much weight to the polarization data, and so the unconstrained fit was repeated one more time, but this time fitting to both the cross-section data and the recoil polarization data from ref. [1] (bottom solid line in fig. 4). Again, the χ^2 increase is significant: χ^2 increases by 65 while the number of degrees of freedom increases by 26 (the number of additional data points). In addition to the fact that the overall fit quality is worse ($\Delta\chi^2_\nu = 0.15$ for ~ 300 degrees of freedom), the recoil polarization data has larger deviations from the global fit than any of the other data sets.

From the above tests we conclude that it is not possible to explain the discrepancy between the two techniques without significant errors in several data sets, or modifications to the cross-section normalization factors that lead to a significantly worse fit. This is not too surprising, as the extractions from single experiments, which do not suffer from uncertainties in the overall normalization, were in agreement with each other but did not agree with the recoil polarization measurements.

5 Conclusions

The discrepancy between electric form factors extracted from recent polarization transfer measurements and older cross-section data implies either a fundamental flaw in one of the techniques, or a problem in one or more experiments. We have shown that the Rosenbluth extractions of G_E from previous measurements are self-consistent, providing that one looks only at analyses that use a single set of data (and after removing the small-angle Walker data). In our new global analysis of the cross-section data, we find no simple explanation for difference between L/T and polarization measurements. A problem with the cross-section measurement would have to be an ϵ-dependent error involving several different data sets.

A problem with either the polarization transfer or L/T data might have significant implications for other experiments using these techniques. If the problems are shown to be in the cross-section data, then the implications are not limited to other L/T measurements. To explain the observed ($\gtrsim 100\%$) inconsistencies in G_E, the cross-section measurements must have ϵ-dependent systematic errors on the scale of a few % or more (which could be rate-dependent, angle-dependent, or energy-dependent errors). While recoil polarization measurements can determine the ratio G_E/G_M, the cross-sections are still needed to extract the individual form factors. Errors of a few % or more in an unknown subset of the cross-section measurements could lead to errors in the form factors at the few % level, and could even imply similar errors in the Q^2-dependence. In addition, elastic cross-section measurements are often used as a benchmark to determine normalization for a variety of measurements.

Until we know which result is correct, we cannot be certain of our knowledge of G_E at $Q^2 > 1$ GeV^2. If E01-001 verifies the polarization transfer measurements, we must still understand the source of the discrepancy in the cross-section data in order to know the form factors to better than the few % level. While it seems likely that such errors would come from cross-section measurements at "extreme" kinematical conditions, and thus would have the largest impact on measurements of the ϵ-dependence, the actual consequence of the discrepancy between the new and old form factor measurements will not be clear until we understand why they disagree.

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